

Interpreting the time of decay measurement: phenomenological significance of the Bohm model

A. S. Majumdar¹

S.N.Bose National Centre for Basic Sciences
Block-JD, Sector III, Salt Lake, Calcutta 700098, India.

Dipankar Home²

Bose Institute, Calcutta 700009, India

Abstract

We point out that the spreading of wave packets could be significant in affecting the analysis of experiments involving the measurement of time of decay. In particular, we discuss a hitherto unexplored application of the Bohm model in properly taking into account the nontrivial effect of wave packet spreading in the CP violation experiment.

PACS No. 03.65.Bz

¹e-mail:archan@boson.bose.res.in

²e-mail:dhom@boseinst.ernet.in

In quantum mechanics, if a particle is represented by a wave packet which evolves according to the Schrodinger equation, an inevitable consequence is the spreading of such a packet. In most experiments the effect of wave packet spreading is negligible in considering the outcome of the measurement. However, there may exist situations where the magnitude of wave packet spreading, in spite of being small, could be nonnegligible when compared to the magnitude of delicate quantum effects being studied. In particular, the observations of minute quantum effects concerning the decay of unstable particles which are inferred from observing the times of decay could be affected by wave packet spreading. This is because the time of decay cannot be measured directly and has to be inferred from appropriate measurements on the decay products whose packets spread before reaching the relevant detectors. Such situations, for example, could occur in verifying CP violation [1], or verifying the departure from exponential decay of unstable nuclei for very short times [2]. In this paper we focus on the CP violation experiment and analyse closely as to what extent the wave packet spreading affects the inference of CP violation from the performed measurements. We argue that, contrary to the usual belief, one *does* require to take into account the wave packet spreading in order to infer CP violation precisely. Our analysis also reveals an interesting application of the Bohm model in quantifying the effect of wave packet spreading in such experiments.

At the outset, let us note that the time of decay is not represented by any hermitian operator. However, in quantum mechanics, any standard measurement is described in terms of hermitian operators. Even for an observable for which there exist no directly corresponding hermitian operator, measurement of it can ultimately be reduced to the measurement of some hermitian operator. For example, when one measures the wavelength of light, one is actually measuring position corresponding to fringe spacing. Other quantities like mass are also finally measured in terms of positions noted from mass spectrograph. The usual textbook justification as to why a nonhermitian operator is not measurable is that its eigenvalues are, in general, complex. However, this is not an adequate argument because the real and/or imaginary components can be taken to correspond to observables. This occurs, for instance, in decay processes where the effective Hamiltonian is complex with its real and imaginary parts corresponding to mass and decay rate respectively. An example of particular relevance is the physics of the nonorthogonal $|K_L >$ (long-lived kaon) and $|K_S >$ (short-lived kaon) states which are eigenstates of a nonhermitian effective Hamiltonian. The experimentally measured distinction between these two states is crucial for the inference of CP-violation in weak interactions of particle physics. On the other hand, there is a theorem based on general quantum mechanical considerations that any measurement of a nonhermitian operator with nonorthogonal eigenstates would lead to superluminal signalling using the EPR-Bohm type nonlocal correlations [3].

Here we recall a simple general argument ruling out the measurability of a nonhermitian operator if a measurement process is describable by linear and unitary quantum mechanics. Let $|\psi_1 >$ and $|\psi_2 >$ be the eigenstates of an observable. The measured state is, say,

$$|\Psi > = a|\psi_1 > + b|\psi_2 > \quad (1)$$

and the apparatus is in an initial state $|A_0\rangle$. Linearity demands that the system-apparatus state $|\Psi\rangle|A_0\rangle$ evolves by measurement interaction as

$$\begin{aligned} a|\psi_1\rangle|A_0\rangle &\rightarrow a|\psi_1\rangle|A_1\rangle \\ b|\psi_2\rangle|A_0\rangle &\rightarrow b|\psi_2\rangle|A_2\rangle \end{aligned} \quad (2)$$

From unitarity, it then follows

$$a^*b\langle\psi_1|\psi_2\rangle = a^*b\langle\psi_1|\psi_2\rangle\langle A_1|A_2\rangle \quad (3)$$

Now, if the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are nonorthogonal eigenstates of a nonhermitian operator, i.e., $\langle\psi_1|\psi_2\rangle \neq 0$, then Eq(3) cannot be satisfied for any measurement in which the two distinguishable states $|A_1\rangle$ and $|A_2\rangle$ are orthogonal. Hence, according to standard quantum mechanics, nonhermitian operators cannot be measured if measurement processes are linear and unitary.

Now, turning to the particular case of measuring the time of decay, it has to be inferred from the measurement of an appropriate hermitian observable. Usually this involves measurement of position/momentum of decay products. In the theory of scattering and decay processes, one describes the decaying particles as well as the decay products in the asymptotic limit by plane waves. However, to be more realistic, one needs to use wave packets instead of plane waves. We shall now argue that the spreading of wave packets in such an experiment involving CP violation gives rise to considerable departures from the predicted coordinates of the decay products.

First, let us recapitulate a few basic features of CP-violation [4]. C(charge conjugation) and P(parity) are two of the fundamental discrete symmetries of nature, the violations of which have not been empirically detected in phenomena other than weak interactions. If a third discrete symmetry T(time reversal) is taken into account, there exists a fundamental theorem of quantum field theory, viz., the CPT theorem which states that all physical processes are invariant under the combined operation of CPT. However, there is no theorem forbidding the violation of CP symmetry. In fact, there have been several experiments to date [5], starting from the pioneering observation of Christenson, Cronin, Fitch and Turlay [4], that have revealed the occurrence of CP violation through weak interactions involving the particles K^0 and \bar{K}^0 . The eigenstates of strangeness K^0 ($s = +1$) and its CP conjugate \bar{K}^0 ($s = -1$) are produced in strong interactions, for example, the decay of Φ particles. Weak interactions do not conserve strangeness, whereby K^0 and \bar{K}^0 can mix through intermediate states like $2\pi, 3\pi, \pi\mu\nu, \pi e\nu$, etc. The observable particles, which are the long lived K -meson K_L , and the short lived one K_S , are linear superpositions of K^0 and \bar{K}^0 , i.e.,

$$|K_L\rangle = (p|K^0\rangle - q|\bar{K}^0\rangle)/\sqrt{|p|^2 + |q|^2} \quad (4)$$

$$|K_S\rangle = (p|K^0\rangle + q|\bar{K}^0\rangle)/\sqrt{|p|^2 + |q|^2} \quad (5)$$

which obey the exponential decay law $|K_L\rangle \rightarrow |K_L\rangle \exp(-\Gamma_L t/2) \exp(-im_L t)$ and analogously for $|K_S\rangle$, where Γ_L and m_L are the decay width and mass respectively of the K_L particle. It follows from (4) and (5) that

$$\langle K_L | K_S \rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} \quad (6)$$

CP violation takes place if the states $|K_L\rangle$ and $|K_S\rangle$ are not orthogonal. Through weak interactions K_S decays rapidly into channels such as $K_S \rightarrow \pi^+\pi^-$ and $K_S \rightarrow 2\pi^0$ with a mean lifetime of $10^{-10}s$, whereas the predominant decay modes of K_L are $K_L \rightarrow \pi^\pm e^\pm \nu$ (with branching ratio $\sim 39\%$), $K_L \rightarrow \pi^\pm \mu^\pm \nu$ ($\sim 27\%$), and $K_L \rightarrow 3\pi$ ($\sim 33\%$) [5]. The CP violating decay mode $K_L \rightarrow 2\pi$ is extremely rare (with branching ratio $\sim 10^{-3}$) in the background of the other large decay modes. The momenta and locations of the emitted pions are important since the key experimental signature is to detect the 2π particles coming from the decay of K_L and identify them as coming from K_L and *not* from K_S .

In a typical experiment to detect CP violation, an initial state of the type

$$|\psi_i\rangle = (a|K_L\rangle + b|K_S\rangle) \quad (7)$$

is used which is a coherent superposition of the K_L and K_S states. Such a state is produced by the technique of ‘regeneration’ [6] which is used in a large number of experiments [5]. The common feature of all these experiments is the measurement of the vector momenta \vec{p}_i of the charged decay products $\pi^+\pi^-$ or $2\pi^0$ from the decaying pions. It is only the *type* of instrument used for actually measuring the momenta that varies from experiment to experiment.

We consider a single event in which the two emitted pions from a decaying kaon are detected by two detectors respectively along two different directions. From the measured momenta \vec{p}_1 and \vec{p}_2 , the *trajectories* followed by the individual pions are *retrodictively* inferred *assuming* that they have followed *classical trajectories*. The point of intersection of these retrodicted *trajectories* is inferred to be the point from which the decay products have originated from the decaying system. In other words, what is technically known as the “decay vertex” is determined in this way. Then momentum of the decaying kaon is obtained by $\vec{p}_k = \vec{p}_1 + \vec{p}_2$. Once the decay vertex and the kaon momentum are known, one estimates the time taken by the decaying kaon to reach the decay vertex from the source, *again* using at this stage the idea of a *classical trajectory*. If this time turns out to be significantly larger than the K_S mean lifetime ($\sim 10^{-10}s$), one infers that the detected 2π pair must have come from K_L and *not* from K_S which, as already mentioned, is the signature of CP violation.

It is thus evident from the above discussion that the assumption of a *classical trajectory* of a freely evolving particle (decaying kaon as well as pion) is a key ingredient in inferring CP violation in such experiments (see also an earlier paper [7], but there the point about the wave packet description and its spreading was not considered). Now, to consider the justification of

such an assumption we note that within the standard interpretation of quantum mechanics, the very concept of a trajectory of a particle is regarded to be inadmissible. One possible argument could be to associate localized wave packets with the emitted pions and decaying kaons, and to use the fact that their peaks follow *classical trajectories* in the case of free evolutions. But then there would be inevitable spreading of these wave packets. It is thus important to consider a quantitative estimate of this behaviour for the experiment discussed here. Let us *quantify* the resulting error or fluctuation due to the spreading of a wave packet by taking into account the *actual distances* involved in the relevant experiments. This is particularly crucial in the present context because the CP violation effect is exceedingly *small* (branching ratio of the CP violating decay mode $K_L \rightarrow 2\pi$ is 10^{-3}). We take dimension of the wave packet of the kaon at the time of its production to be typically of the order of $1f$, i.e., its initial spread $\sigma_0 \approx 10^{-13}cm$. After a time t , its spread σ is given by

$$\sigma = \sigma_0 \left(1 + \left(\frac{\hbar t}{2m\sigma_0^2} \right)^2 \right)^{1/2} \quad (8)$$

After the kaon travels about $300K_S$ decay lengths ($\approx 10m$), which is the usual distance involved in the relevant experiments, one obtains $\sigma = 10^5m$ – an astonishingly large number ! One may attribute this to the nonrelativistic nature of the above analysis. However, the prediction of CP violation in the relevant experiments can be formally described adequately in terms of the Schroedinger equation (see [1] and references therein). Nevertheless, even if relativistic corrections alter the value of σ by several orders of magnitude, σ would still be too large for unambiguously inferring CP violation in such experiments. Surprisingly, in the analysis of *none* of the CP violation experiments performed to date, this point has been considered.

In order to take into account such an effect of wave packet spreading, the Bohm model (BM) can play a useful role in estimating accurately the position of decay vertex from the observed position/momentum of decay products. To explain this, we first briefly recapitulate the key elements of BM. BM provides an ontological and self-consistent interpretation of the formalism of quantum mechanics in terms of *particle trajectories* [8,9,10]. Predictions of BM are in agreement with that of standard quantum mechanics. In BM a wave function ψ is taken to be an incomplete specification of the state of an individual particle. An objectively real “position” coordinate (“position” existing irrespective of any external observation) is ascribed to a particle apart from the wave function. Its “position” evolves with time obeying an equation that can be derived from the Schroedinger equation (considering the one dimensional case)

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \equiv -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi \quad (9)$$

by writing

$$\psi = Re^{iS/\hbar} \quad (10)$$

and using the continuity equation

$$\frac{\partial}{\partial x}(\rho v) + \frac{\partial \rho}{\partial t} = 0 \quad (11)$$

for the probability distribution $\rho(x, t)$ given by

$$\rho = |\psi|^2. \quad (12)$$

It is important to note that ρ is ascribed an *ontological* significance by regarding it as representing the probability density of “particles” occupying *actual* positions. On the other hand, in the standard interpretation, ρ is interpreted as the probability density of *finding* particles around certain positions. Setting (ρv) equal to the quantum probability current leads to the Bohmian equation of motion where the particle velocity $v(x, t)$ is given by

$$v \equiv \frac{dx}{dt} = \frac{1}{m} \frac{\partial S}{\partial x} \quad (13)$$

The particle trajectory is thus deterministic and is obtained by integrating (13) for a given initial position.

Now let us examine the nature of Bohmian trajectories in the case of a wave packet. An ensemble of particles distributed over a wave packet possess different ontological positions. It can be shown that particles initially at the centre of the wave packet follow classical trajectories [9]. But all particles with initial positions away from the centre of the wave packet follow *nonclassical* Bohmian trajectories. The position of any particle at a time t_2 , denoted by $X(t_2)$, can be computed given its initial position X_0 and velocity v_0 at time t_1 . The magnitude of departure from classical trajectories is embodied in the second term of the following relation (see Ref. [9])

$$X(t_2) = v_0(t_2 - t_1) + X_0 \left(1 + \left(\frac{\hbar(t_2 - t_1)}{2m\sigma_0^2} \right)^2 \right)^{1/2} \quad (14)$$

In any experiment involving the measurement of time of decay, in particular, in the CP violation experiment, one could measure the set of times at which the vector momenta of the decay products are recorded by a set of detectors located at various positions. In terms of Eq.(14), the values of $X(t_2)$ are recorded for various t_2 . Then inverting Eq.(14), it is possible to express X_0 as a function of t_1 for any particular t_2 . Using the whole set of such data this exercise can be used to obtain an ensemble of retrodicted trajectories for the decay products. The various intersection points of these retrodicted trajectories correspond to various decay vertices of the corresponding subensembles of trajectories. In this way one can estimate a spread of decay vertices for the kaons. From this, it is possible to obtain the spread in decay times for the kaons by using the trajectory equation for kaons, knowing their origins.

We have thus shown that using BM one can *retrodict* the trajectories for the decay products and decaying kaons, given the measured vector momenta and arrival times of the decay products. This would then enable to calculate the spread in decay times for the decaying kaons. The *nonclassical* second term in the trajectory equation(14) is crucial in calculating this spread. On the other hand, within the standard framework, the best one can do is to describe the propagation of the peak of a wave packet by the classical equation. In the absence of any equation of motion for trajectories within standard quantum mechanics, it then remains a nontrivial issue to take into account consistently the effect of wave packet spreading for estimating accurately *when* the kaons have decayed into 2π pairs that originate only from K_L and not from K_S . BM turns out to provide a convenient scheme for addressing this issue. It should be worthwhile to look for more such examples where BM can usefully supplement the standard framework for analysing the experimental results with more precision. An interesting area for such a study would be to analyse in terms of Bohmian trajectories the recently claimed experimental detection of minute departure from the exponential behaviour in quantum tunnelling [11] which involves essentially the measurement of decay times.

The research of DH is supported by the Department of Science and Technology, Govt. of India.

REFERENCES

- [1] For a review, see for instance, K.Kleinknecht, in “CP violation”, edited by C.Jarlskog, (World Scientific, Singapore, 1989) pp. 41 -104.
- [2] T. D. Nghiep, V. T. Hanh and N. N. Son, Nucl. Phys. B (Proc. Suppl.) **66** (1998) 533.
- [3] E. J. Squires, Phys. Lett. A **130**, 192 (1988).
- [4] J.H.Christenson, J.W.Cronin, V.L.Fitch and R.Turlay, Phys. Rev. Lett. **13**, 138 (1964).
- [5] For eample, see, C.Geweniger et al., Phys. Lett. B **48** (1974) 487; V.Chaloupka et al., Phys. Lett. B **50** (1974) 1; W.C.Carithers et al., Phys. Rev. Lett. **34** (1975) 1244; N.Grossmann, et al., Phys. Rev. Lett., **59** (1987) 18.
- [6] A.Pais and O.Piccioni, Phys. Rev. **100** (1955) 1487.
- [7] D. Home and A. S. Majumdar, Found. Phys. **29**, 721 (1999).
- [8] D.Bohm, Phys. Rev. **85** (1952) 166; D.Bohm and B.J.Hiley, “The Undivided Universe”, (Routledge, London, 1993).
- [9] P.R.Holland, “The Quantum Theory of Motion”, (Cambridge University Press, London, 1993).
- [10] J.T.Cushing, “Quantum Mechanics - Historical Contingency and the Copenhagen Hegemony”, (University of Chicago Press, Chicago, 1994); D. Home, “Conceptual Foundations of Quantum Physics” (Plenum, NY, 1997).
- [11] S. R. Wilkinson et al., Nature **387** (1997) 575.